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$$r_1 r_2 r_3 \ldots r_n = \tfrac{1}{n}[R_1 S_{n-1} - R_2 S_{n-2} + \ldots + (-1)^{n-2} R_{n-} \; S_1 + (-1)^{n-1} R_n].$$

$n = 2, 3, 4,$ &c.,

$$r_1 r_2 = \tfrac{1}{2}[(r_1 + r_2)^2 - r_1^2 - r_2^2]; \; r_1 r_2 r_3 = \tfrac{1}{3}[\tfrac{1}{2}(r_1 + r_2 + r_3)^3 - \tfrac{1}{2}(r_1 + r_2 + r_3)(r_1^2 + r_2^2 + r_3^2) - (r_1 + r_2 + r_3)(r_1^2 + r_2^2 + r_3^2) + r_1^3 + r_2^3 + r_3^3] = \tfrac{1}{3}[\tfrac{1}{2}(r_1 + r_2 + r_3)^3 - \tfrac{3}{2}(r_1 + r_2 + r_3)(r_1^2 + r_2^2 + {}^{2}_{3}) + r_1^3 + r_2^3 + r_3^3].$$

These transformations show how easily we may derive results by a general process, that would otherwise be difficult.

* * *

DEMONSTRATION OF TWO OF PROF. PEIRCE'S PROPOSITIONS.

BY PROF. M. L. COMSTOCK, OF KNOX COLLEGE, ILL.

In the *Mathematical Montlity* (Runkle's), No. 1, Vol. 1, Prof. Peirce gave a set of "Propositions on the Distribution of Points on a Line." They were subsequently solved by Prof. W. P. G. Bartlett and R. J. Adcock. I send you a solution of two of them by a method presumably applicable to the others.

Prop. 1.—"If two points are taken in all possible relative positions, upon a given line, their distance apart in one-half the whole number of possible positions, is less than .29289 of the length of the line."

Prop. 2.—"The number of positions in which the distance between the points is less than half the length of the line is $\frac{3}{4}$ of the whole number of possible positions."

If $a =$ the length of the line, and $x =$ the shortest distance between two points, it readily appears that

$\frac{a}{x} =$	number of	positions at	the distance	x,
$\frac{a}{x} - 1 =$	"	"	"	$2x$.
$\frac{a}{x} - 2 =$	"	"	"	$3x$,
$\vdots$				
$2 =$	"	"	"	$a - x$,
$1 =$	"	"	"	a,

Hence $\frac{a}{2x}(\frac{a}{x} + 1)$ = whole number of possible positions, since these quantities are in arithmetical progression, and the number of terms is $a \div x$.

Notice also that, if x is taken as the measuring unit, the number of the term is the same as the distance between the points in that term.

Now let z = the number of terms which shall embrace half ot an arithmetical reries whose first term is t, last term 1, com. dif. 1, and the number of terms n.

The sum embraced by z will be

$$\tfrac{1}{2}z(2t - z + 1);$$

also, $t - z$ is the first term of what remains of the given series, 1 is the last term, and $n - z$ the number of terms; hence the sum is

$$\tfrac{1}{2}(n - z)(t - z + 1),$$

$$\tfrac{1}{2}z(2t - z + 1) = \tfrac{1}{2}(n - z)(t - z + 1),$$

$$\text{and } z^2 - \tfrac{1}{2}(3t + 2 + n)\, z = - \tfrac{1}{2}(z + 1)\, n.$$

Now in our example,

$a \div x$= number of positions at the distance $x = t$,

$a \div x = n$, as readily appears in our series, hence,

$$z^2 - \left(2\frac{a}{x} + 1\right)z = - \frac{1}{2}\left(\frac{a^2}{x^2} + \frac{a}{x}\right);$$

and making x infinitely small,

$$z^2 - \frac{2a}{x}\, z = - \frac{1}{2}\,\frac{a^2}{x^2},\ z = \left(1 - \frac{\sqrt{2}}{2}\right)\frac{a}{x} = (.29289 -)\frac{a}{x},$$

which establishes Prop. 1, x being called the measuring unit.

Again, if we place the points at a distance from each other equal to half the line we shall have

$\frac{a}{2x} + 1$ = number of positions at the distance $\frac{a}{2}$,

$\frac{a}{2x}$ = “ “ “ “ $\frac{a}{2} + x$,

$\frac{a}{2x} - 1$ = “ “ “ “ $\frac{a}{2} + 2x$,

⋮

2 = “ “ “ “ a -- x,

1 = “ “ “ “ a.

Here the number of terms is $\frac{a}{2x} + 1$, and the sum

$= \frac{1}{2}\left(\frac{a}{2x} + 1\right)\left(\frac{a}{2x} + 2\right) =$ number of positions in which the distance between the points is *not less* than half the line.

But the whole number of positions as seen before is $\frac{a}{2x^2}\,(a + x)$.

When x is taken very small, these quantities reduce to $\frac{a^2}{8x^2}$ and $\frac{a^2}{2x^2}$, the first part of which is $\frac{1}{4}$ the second, hence the whole number of positions in which the greatest distance is *less* than half the line is $\frac{3}{4}$ of the whole number of possible positions.

SOLUTIONS OF PROBLEMS IN NO. 7, AND 29, IN NO. 6.

Solutions of problems in No. 7 have been received as follows:

From Geo. L. Dake, 30, 31 & 32; Geo. M. Day, 30, 32 & 33; Prof. A. B. Evans, 30, 31, 32 & 33; Prof. H. T. Eddy, 32; Henry Gunder, 30, 31, 32 & 33; H. Heaton, 30, 31, 32 & 33; Prof. E. W. Hyde, 32 & 33; Prof. D. Kirkwood, 31; Artemas Martin, 30, 31, 32 & 33; L. Regan, 30 & 31; Walter Siverly, 30, 31, 32 & 33; Werner Stille, 30, 31, 32 & 33; E. B. Seitz, 30. 31, 32 & 33; Prof. D. Trowbridge, 30, 31 & 32.

Elegant solutions of all the questions proposed in No. 6 were received in due time, last month, from Walter Siverly, but the letter containing his solutions of 25, 26 and 29 was mis-laid and hence proper credit was not given him in No. 8. Mr. Martin also sent correct solutions of Nos. 25 and 26, but failed to get credit for them in No. 8, because his letter, also, was mis-laid.

29. "If $a, b, c, d, e, f, g, h, i, j, k$ be chords drawn from any point on the circumference of a circle to the eleven angles of an inscribed regular polygon of eleven sides; prove that

$(a + k)(b + j)(c + i)(d + h)(e + g) = f^5$..................(1)."